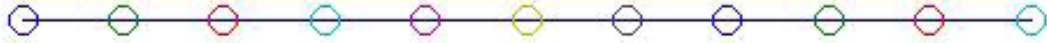


## Introduction to Seismic Waves Fog 2 Lab (Dueker 3/20/05)

Name \_\_\_\_\_ Lab and HW due Friday 4/1 (no class next Mon, Weds, Thurs)

Consider a set of masses ( $m$ ) connected by springs with strength ( $k$ ). We can think of the masses as atoms and the spring as the bonds between them. The masses are drawn as the equally spaced circles in their rest position below.



We want to find a function of space and time that describes the displacement of the masses from their equilibrium (rest) position. Let us call this function  $u(x,t)$  where  $x$  is the distance along the  $x$ -axis and  $t$  is time.

Let's consider the forces acting on the masses when the set of masses is set in motion by a disturbance (e.g., earthquake or your voice). This is simple as there are only two forces to consider in this simplified 1-dimensional example (e.g., we are ignoring friction).

### THE TWO FORCES

1. Any movement of an atom from its rest position will incur a push and pull force on the two bonds that connect each atom to its two surrounding atoms. We call this the elastic restoring force and it is proportional to a material stiffness ( $k$ ) times the spatial derivative of the stress which is just the spatially curvature of the displacement function  $u(x,t)$

$$F_E = k \frac{\partial^2 u(x,t)}{\partial x^2}.$$

2. Any atom that is moving from its rest position will be undergoing accelerations which creates an inertial force. This is just Newton's second law

$$F_I = ma = m \frac{\partial^2 u(x,t)}{\partial t^2}.$$

Now, let's balance the forces by equating the two force terms to get the "wave equation":

$$m \frac{\partial^2 u(x,t)}{\partial t^2} = k \frac{\partial^2 u(x,t)}{\partial x^2} \quad \text{where } m \text{ is mass and } k \text{ is bond strength.}$$

We can rearrange this equation as:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{k}{m} \frac{\partial^2 u(x,t)}{\partial x^2}.$$

Remember this because later you will show that  $k/m$  will be the definition of the waves velocity squared.

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1. Demonstrate that a general solution to the wave equations is just  $u(x,t) = f(x+vt)$  and that this general solutions solves the wave equation if  $v^2 = k/m$ . Do this by calculating the second space and time derivatives of the function and then plugging these results into the wave equation. After algebraically simplifying you should end up showing that  $v = \lambda * f$ . To get things going, the second spatial derivative is calculated; Note the use of the change rule is required.

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial}{\partial x} \left[ \frac{\partial u(x,t)}{\partial x} \right] = \frac{\partial}{\partial x} \left[ \frac{\partial f(x,t)}{\partial x} \right] = \frac{\partial}{\partial x} \left[ \frac{\partial f}{\partial x} \frac{\partial(x-vt)}{\partial x} \right] = \frac{\partial}{\partial x} \left[ \frac{\partial f}{\partial x} \right] = \frac{\partial^2 f}{\partial x^2}$$

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial}{\partial t} \left[ \frac{\partial u(x,t)}{\partial t} \right] =$$

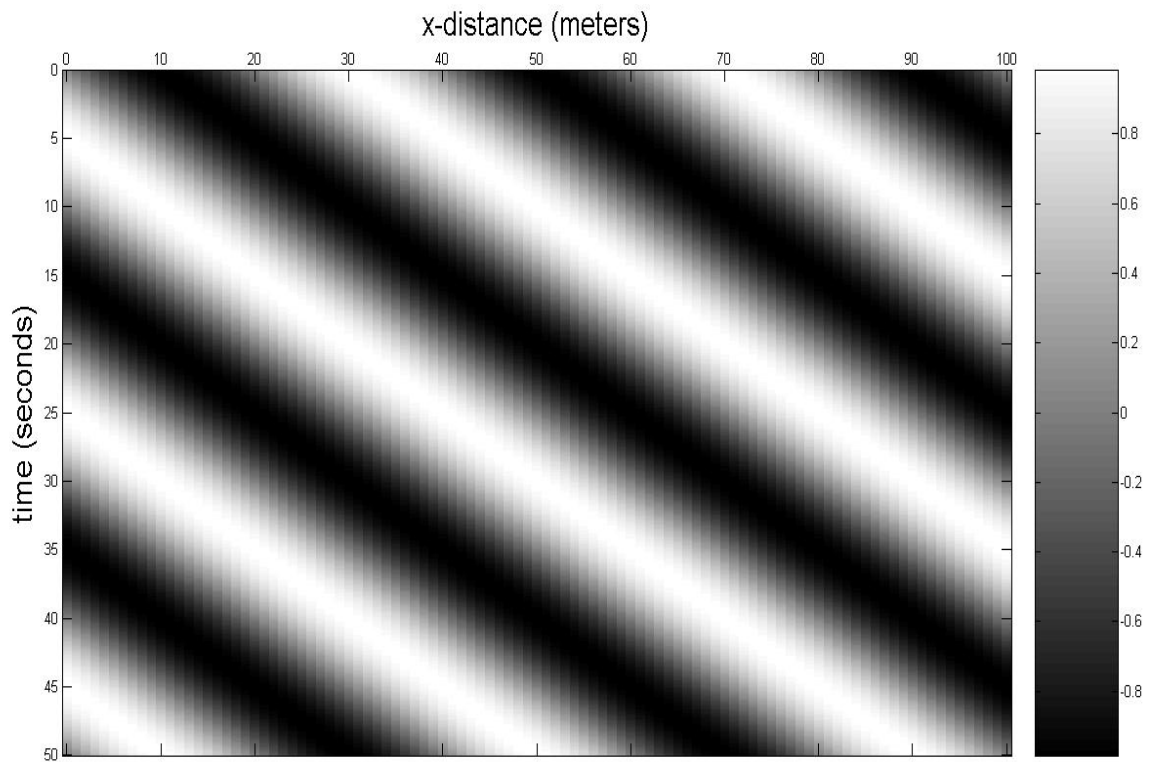
2. Assume a steady-state wave solution that is just a cosine wave:

$$u(x,t) = \cos\left(2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right) \quad \text{where } T \text{ and } \lambda \text{ are the wave's period and wavelength.}$$

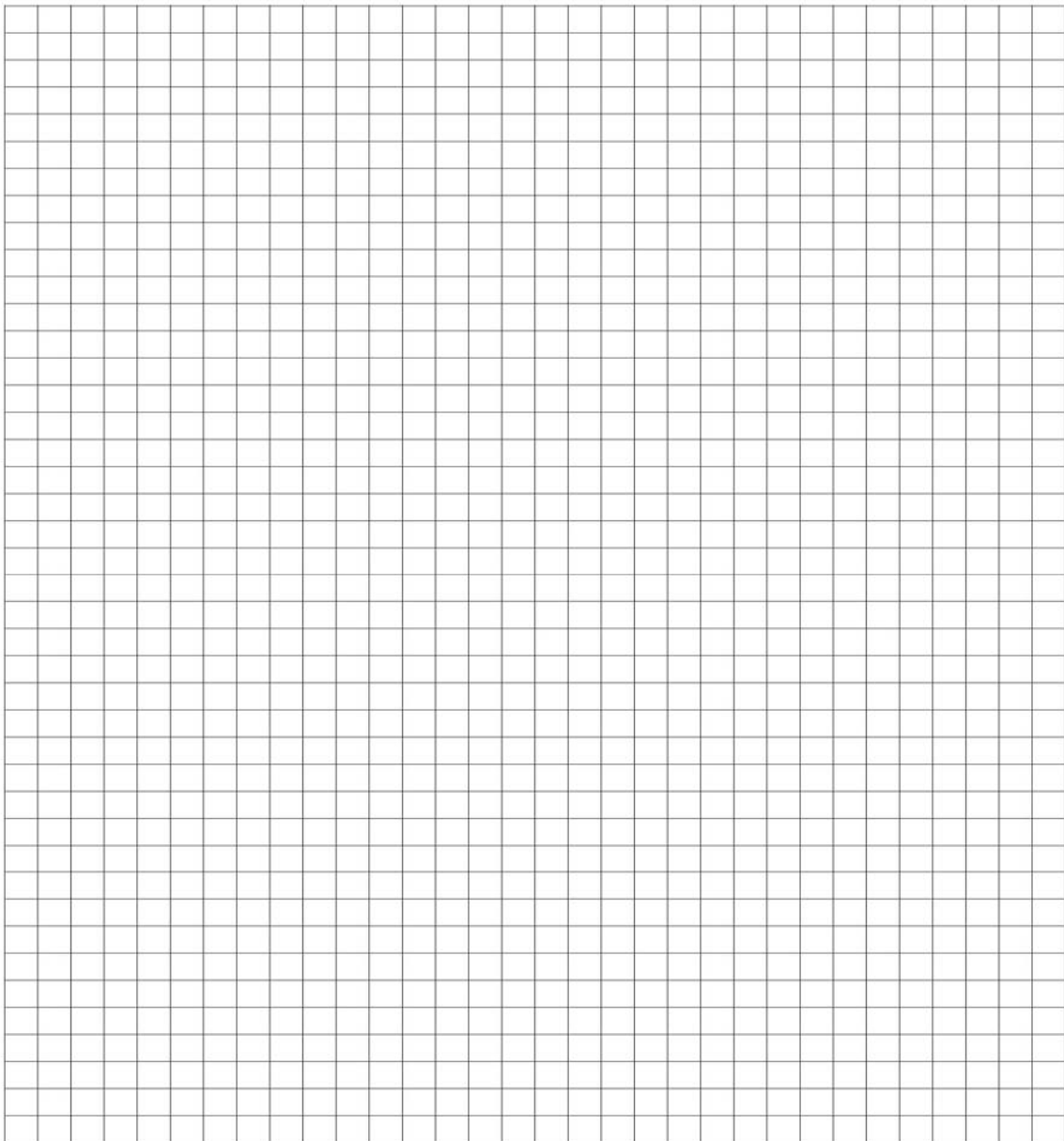
Show that this cosine function is a valid solution to the wave equation by taking the second spatial and time derivatives and then substituting these terms into the wave equation and algebraically simplifying.

3. Below is a space-time graph of the cosine wave solution used in Problem 2.

- (a) Draw lines to show where the peaks (+1 amplitude) and troughs (-1 amplitude) are.
- (b) Measure and report and label what the period and wavelength of the cosine wave is.
- (b) Calculate the velocity using the relation  $v = \lambda f$  and confirm this by measuring the velocity directly from the graph.



4. Let's pretend we are an observer watching the cosine wavefield.
- (a) Assume you are sitting at the position  $x=30$  meters. Graph the wavefield amplitude that you would see from 0-50 seconds.
  - (b) Assuming that time is stopped at  $t=20$  seconds. Graph the wavefield amplitude that you see from 0-100 meters.
  - (c) Assume at time=0, you are sitting at  $x=30$  meters AND you are moving at the velocity of the wave (you calculated previously). Graph the wavefield amplitude you would experience over the time interval from 0-40 seconds.



5. FOR NEXT WEEK WHILE I AM GONE. Goto the following website:  
<http://www.visualentities.com/applets/earthvelocity.htm>. Note that your browser will need to have Java enable to see the applet (it should unless you a working on a real old computer). The help tab on the webpage tells you how to change velocity model.

(a) Create a P- and S-wave velocity model that matches the observed P- and S-travel-times.

(b) Take a screen shot of your final model with the “times tab” subplot displayed and turn this in with the lab. For a windows OS, use the “PRT-SC” button that will take screen shot and put it on the clipboard.